بسم الله الرحمن الرحيم

King Abdulaziz University

Review 2 for The First Exam-Fall 2013

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- أنقر على Start لبداء الاختبار.
- يحتوي هذا الأختبار على 41 سؤالاً.
- عند الانتهاء من الاختبار أنقر على End للحصول على النتيجة.
- بالتوفيق إن شاء اللهو كل عام وانت بخير.

Calculus II
Math202

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Enter Name:

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Start Answer each of the following.

1. I $\sinh^{-1} x =$

- $\ln\left(x+\sqrt{x^2-1}\right)$
- $\ln\left(x-\sqrt{x^2-1}\right)$
- $\ln\left(x+\sqrt{x^2+1}\right)$
- $\ln\left(x-\sqrt{x^2+1}\right)$

2. $\frac{1+\tanh x}{1-\tanh x} =$

- 1
- e^x
- e^{2x}
- e^{-x}

3. If $\tanh x = \frac{12}{13}$, then $\cosh x =$

- $\frac{12}{5}$
- $\frac{12}{13}$
- $\frac{13}{5}$
- $\frac{5}{13}$

4. If $y = \tan^{-1}(\tanh x)$, then $y' =$

- ☐ e^{2x}
- ☐ $\sinh(2x)$
- ☐ $\cosh(2x)$
- ☐ $\operatorname{sech}(2x)$

5. $\lim_{x\to\infty} \frac{\sinh x}{e^x}$.

- ☐ 2
- ☐ e^2
- ☐ $e^{\frac{1}{2}}$
- ☐ $\frac{1}{2}$

6. If $y = \sinh^{-1}(\tan x)$, then $y' =$

- ☐ $\sec x$
- ☐ $-\sec x$
- ☐ $|\sec x|$
- ☐ $\tan x$

7. If $y = \operatorname{sech}^{-1}(e^x)$, then $y' =$

☐ $\frac{-1}{\sqrt{1-e^{2x}}}$ ☐ $\frac{1}{\sqrt{1-e^{2x}}}$

☐ $\frac{-1}{\sqrt{1+e^{2x}}}$ ☐ $\frac{1}{\sqrt{1+e^{2x}}}$

8. If $y = \cosh(\ln x)$, then $y' =$

☐ $\cosh\left(\frac{1}{x}\right)$ ☐ $\frac{\sinh(\ln x)}{x}$

☐ $\frac{\cosh(\ln x)}{x}$ ☐ $\sinh\left(\frac{1}{x}\right)$

9. $\lim_{x\to\infty}\left[\ln x - \sinh^{-1} x\right] =$

☐ 1 ☐ $-\ln 2$

☐ 0 ☐ $\ln 2$

10. $\lim_{x\to 0^+}\left(\frac{\cos(3x)}{\cos(2x)}\right)^{\frac{1}{x^2}} =$

- ☐ 1
- ☐ $\frac{-5}{2}$
- ☐ $\frac{1}{\sqrt{e^5}}$
- ☐ $\sqrt{e^5}$

11. $\lim_{x\to 0}\frac{\sin(4x)}{\sin(8x)} =$

- ☐ 0
- ☐ $\frac{1}{2}$
- ☐ 1
- ☐ 2

12. $\lim_{x\to 0}\frac{2^x-1}{x} =$

- ☐ 0
- ☐ $\frac{1}{2}$
- ☐ 1
- ☐ 2

13. If $f''(\theta) = \sin\theta + \cos\theta$ and $f(0) = 3, f'(0) = 4$, then $f(\theta) =$

- ☐ $-\sin\theta + \cos\theta + 5\theta + 4$
- ☐ $-\sin\theta - \cos\theta + 5\theta + 4$
- ☐ $-\sin\theta - \cos\theta - 5\theta + 4$
- ☐ $\sin\theta - \cos\theta + 5\theta + 4$

14. If $f''(x) = 4 + 6x + 24x^2$, $f(0) = 3$, and $f(1) = 10$ then $f(x) =$

- ☐ $2x^4 + x^3 - 2x^2 + 2x + 3$
- ☐ $x^4 + 2x^3 + 3x^2 + x + 3$
- ☐ $2x^4 + x^3 + 2x^2 + 2x + 3$
- ☐ $x^4 + x^3 + 2x^2 + 2x + 3$

15. The sigma notation of $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+\frac{1}{36}$ is

- ☐ $\sum_{i=1}^{5}\frac{1}{(i+1)^2}$
- ☐ $\sum_{i=1}^{5}\frac{1}{i}$
- ☐ $\sum_{i=1}^{6}\frac{1}{i^2}$
- ☐ $\sum_{i=1}^{5}\frac{1}{i^2}$

16. $\sum_{j=0}^{4}\left(2^j+j^2\right)=$

- ☐ 59
- ☐ 58
- ☐ 60
- ☐ 61

17. An estimation of the area under the graph $y=x^2+1$ from $x=-1$ to $x=2$ using three rectangles and right end -points is

- ☐ 1
- ☐ 7
- ☐ 8
- ☐ 3

18. Express the limit $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{x_i^*}{(x_i^*)^2+4}\Delta x$ as a definite integral on the interval $[1,3]$

- ☐ $\int_1^3 \frac{x}{x^2+4}\,dx$
- ☐ $\int_1^3 \frac{1}{x^2+4}\,dx$
- ☐ $\int_1^3 \frac{x}{x+4}\,dx$
- ☐ $\int_1^3 \frac{x^2}{x+4}\,dx$

19. If $\int_{-2}^{1} f(x)\,dx = 30$ and $\int_{-2}^{-1} f(x)\,dx = 10$, then $\int_{-1}^{1}\left[\frac{3\sin x}{1+x^2}+4f(x)\right]dx$

- ☐ 140
- ☐ 40
- ☐ 80
- ☐ 20

20. If $h(x)=\int_1^{e^x}\ln t\,dt$, then $h'(x)=$

- ☐ xe^x
- ☐ e^{2x}
- ☐ e^x
- ☐ $e^x\ln x$

21. $\int_1^2 \frac{x^3+3x^6}{x^4}\,dx =$

- ☐ 7
- ☐ 8
- ☐ $7+\ln 2$
- ☐ $8+\ln 2$

22. $\int_1^{18} \sqrt{\frac{3}{x}}\,dx =$

- ☐ 16
- ☐ $6\sqrt{6}-3\sqrt{2}$
- ☐ $6\sqrt{6}-2\sqrt{3}$
- ☐ $6\sqrt{6}-3\sqrt{2}$

23. If $\int_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}} \frac{4}{\sqrt{1-x^2}}\,dx =$

- ☐ π
- ☐ $\frac{\pi}{3}$
- ☐ $\frac{\pi}{4}$
- ☐ $\frac{\pi}{2}$

24. If $\int (x+4)(2x+1)\,dx =$

- ☐ $\frac{1}{3}x^3 + \frac{9}{2}x^2 + 4x + C$
- ☐ $x^3 + x^2 + 4x + C$
- ☐ $(\frac{1}{2}x^2 + 4x)(x^2 + x) + C$
- ☐ $\frac{2}{3}x^3 + \frac{9}{2}x^2 + 4x + C$

25. If $\int [1 + \tan^2 \alpha]\,d\alpha =$

- ☐ $\alpha - \sec\alpha + C$
- ☐ $\alpha + \sec\alpha + C$
- ☐ $\tan\alpha + C$
- ☐ $\sec\alpha \tan\alpha + C$

26. $\int [10^x + x^{10}]\,dx =$

- ☐ $\frac{11^x}{11} + \frac{x^{11}}{11} + C$
- ☐ $\frac{10^x}{\ln 10} + \frac{x^{11}}{11} + C$
- ☐ $\frac{10^{x+1}}{x+1} + \frac{x^{11}}{11} + C$
- ☐ $10^x \ln 10 + \frac{x^{11}}{11} + C$

27. $\int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2}\,dx =$

- ☐ $\sec^2\left(\frac{1}{x}\right) + C$
- ☐ $\tan\left(\frac{1}{x}\right) + C$
- ☐ $\sec\left(\frac{1}{x}\right)\tan\left(\frac{1}{x}\right) + C$
- ☐ $\sec\left(\frac{1}{x}\right) + C$

28. $\int \frac{1}{\cos^2 t\sqrt{1+\tan t}}\, dt =$

- ☐ $\sqrt{1+\tan t}+C$
- ☐ $\frac{1}{\sqrt{1+\tan t}}+C$
- ☐ $2\sqrt{1+\tan t}+C$
- ☐ $\ln\left|\sqrt{1+\tan t}\right|+C$

29. $\int \frac{\sin(2x)}{1+\cos^2 x}\, dx =$

- ☐ $-\tan^{-1}(\cos x)+C$
- ☐ $\ln(1+\cos^2 x)+C$
- ☐ $\frac{-1}{1+\cos^2 x}+C$
- ☐ $-\ln(1+\cos^2 x)+C$

30. $\int \frac{\sin x}{1+\cos^2 x}\, dx =$

- ☐ $-\tan^{-1}(\cos x) + C$
- ☐ $\ln(1 + \cos^2 x) + C$
- ☐ $\frac{-1}{1+\cos^2 x} + C$
- ☐ $-\ln(1 + \cos^2 x) + C$

31. $\int \frac{1}{\sqrt{1-x^2}\sin^{-1} x}\, dx =$

- ☐ $-\tan^{-1}(\sqrt{1 - x^2}) + C$
- ☐ $\ln(\sqrt{1 - x^2}) + C$
- ☐ $\frac{-1}{\sqrt{1-x^2}} + C$
- ☐ $\ln|\sin^{-1} x| + C$

32. $\int 5^x \sin(5^x)\, dx =$

- ☐ $\frac{\cos(5^x)}{\ln 5} + C$
- ☐ $-\ln 5 \cos(5^x) + C$
- ☐ $\frac{-\cos(5^x)}{\ln 5} + C$
- ☐ $\sin(5^x \ln 5) + C$

33. $\int \frac{x^2}{1+x^6}\, dx =$

- ☐ $\tan^{-1}(x^3) + C$
- ☐ $\ln(1 + x^6) + C$
- ☐ $(1 + x^6)^2 + C$
- ☐ $\sin^{-1}(x^3) + C$

34. $\int \sin x \sec^2(\cos x)\, dx =$

- ☐ $-\tan(\sin x) + C$
- ☐ $\sec(\cos x)\tan(\cos x) + C$
- ☐ $\sec^2(\sin x) + C$
- ☐ $-\tan(\cos x) + C$

35. $\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}\, dx =$

- ☐ $\tan^{-1}(\sec x) + C$
- ☐ $\sin^{-1}(\tan x) + C$
- ☐ $\sqrt{1-\tan^2 x} + C$
- ☐ $\sec^{-1}(\tan x) + C$

36. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx =$

- ☐ $\frac{e^{\sqrt{x}}}{2} + C$
- ☐ $\frac{2}{3}e^{\sqrt{x^3}} + C$
- ☐ $e^{\sqrt{x}} + C$
- ☐ $2e^{\sqrt{x}} + C$

37. $\int x^3\sqrt{x^2+1}\,dx =$

- ☐ $\frac{1}{5}(x^2+1)^{5/2} - \frac{1}{3}(x^2+1)^{3/2} + C$
- ☐ $\frac{2}{3}(x^2+1)^{3/2} + C$
- ☐ $\frac{1}{5}(x^2+1)^{5/2} + \frac{1}{3}(x^2+1)^{3/2} + C$
- ☐ $\sqrt{x^2+1} + C$

38. $\int_0^1 \frac{1}{(1+\sqrt{x})^4}\,dx =$

- ☐ $\frac{1}{6}$
- ☐ $\frac{2}{3}$
- ☐ $\frac{13}{24}$
- ☐ $\frac{11}{24}$

39. $\int \frac{1+x}{1+x^2}\,dx =$

- ☐ $\sqrt{1+x^2}+C$
- ☐ $\tan^{-1}x+C$
- ☐ $\ln(1+x^2)+C$
- ☐ $\tan^{-1}x+\ln(\sqrt{1+x^2})+C$

40. $\int \frac{3^x}{1+3^x}\,dx =$

- $\frac{\ln(1+3^x)}{\ln 3} + C$
- $\ln\left(\frac{1+3^x}{3}\right) + C$
- $\ln(1+3^x) + C$
- $\tan^{-1}(3^x) + C$

41. $\int \frac{3^x}{1+3^{2x}}\,dx =$

- $\frac{\ln(1+3^x)}{\ln 3} + C$
- $\ln\left(\frac{1+3^x}{3}\right) + C$
- $\ln(1+3^x) + C$
- $\frac{\tan^{-1}(3^x)}{\ln 3} + C$

End

Score: Correct

Answers:

Points: Percent:

Letter Grade:

Solutions to Quizzes

Solution to 1. You should remember that $\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right)$. ■

Solution to 2. Since $\cosh x + \sinh x = e^x$ and $\cosh x - \sinh x = e^{-x}$ then

$$\frac{1+\tanh x}{1-\tanh x} = \frac{1+\frac{\sinh x}{\cosh x}}{1-\frac{\sinh x}{\cosh x}} \frac{\cosh x}{\cosh x}$$

$$= \frac{\cosh x + \sinh x}{\cosh x - \sinh x}$$

$$= \frac{e^x}{e^{-x}} = e^{2x}$$

■

Solution to 3. Since $\operatorname{sech}^2 x = 1 - \tanh^2 x$ then $\operatorname{sech}^2 x = 1 - \left(\frac{12}{13}\right)^2 = 1 - \frac{144}{169} = \frac{169-144}{169} = \frac{25}{169}$. Hence $\operatorname{sech} x = \sqrt{\frac{25}{169}} = \frac{5}{13}$ since $\operatorname{sech} x > 0$. Therefore $\cosh x = \frac{1}{\operatorname{sech} x} = \frac{13}{5}$. ■

Solution to 4. Remember that $\cosh^2 x + \sinh^2 x = \cosh(2x)$.

$$\begin{aligned} y &= \tan^{-1}(\tanh x) \\ y' &= \frac{(\tanh x)'}{1+(\tanh x)^2} \\ &= \frac{\operatorname{sech}^2 x}{1+\tanh^2 x} \\ &= \frac{1-\tanh^2 x}{1+\tanh^2 x}\,\frac{\cosh^2 x}{\cosh^2 x} \\ &= \frac{\cosh^2 x - \sinh^2 x}{\cosh^2 x + \sinh^2 x} \\ &= \frac{1}{\cosh(2x)} = \operatorname{sech}^2(2x). \end{aligned}$$

■

Solution to 5.

$$\lim_{x\to\infty} \frac{\sinh x}{e^x}$$ Direct substitution will give I.F. type ∞/∞.

$$= \lim_{x\to\infty} \frac{\sinh x}{e^x}$$ Since $(\sinh x)' = \cosh x, (e^x)' = e^x$,

L.R. will not help. Use the def. of $\sinh x$.

$$= \lim_{x\to\infty} \frac{\frac{e^x - e^{-x}}{2}}{e^x}$$ Since $\sinh x = \dfrac{e^x - e^{-x}}{2}$.

$$= \lim_{x\to\infty} \left[\frac{e^x}{2e^x} - \frac{e^{-x}}{2e^x}\right]$$ Since $\dfrac{a-b}{c} = \dfrac{a}{c} - \dfrac{b}{c}$.

$$= \lim_{x\to\infty} \left[\frac{1}{2} - \frac{e^{-2x}}{2}\right] = \frac{1}{2}$$ Since $\dfrac{1}{e^x} = e^{-x}$, and $\lim_{x\to\infty} e^{-2x} = 0$.

■

Solution to 6.

$$y = \sinh^{-1}(\tan x)_{\text{use the fact } (\sinh^{-1}(f(x)))' = \frac{f'(x)}{\sqrt{1+[f(x)]^2}}}$$

$$\begin{aligned} y' &= \frac{\sec^2 x}{\sqrt{1+\tan^2 x}} \\ &= \frac{\sec^2 x}{\sqrt{\sec^2 x}} \\ &= \frac{\sec^2 x}{|\sec x|} \\ y' &= |\sec x| \end{aligned}$$

■

Solution to 7.

$$y = \operatorname{sech}^{-1}(e^x) \qquad \text{use the fact } \left(\operatorname{sech}^{-1}(f(x))\right)' = \frac{-f'(x)}{f(x)\sqrt{1-[f(x)]^2}}$$

$$y' = \frac{-e^x}{e^x\sqrt{1+(e^x)^2}}$$

$$= \frac{-e^x}{e^x\sqrt{1-e^{2x}}}$$

$$y' = \frac{-1}{\sqrt{1-e^{2x}}}$$

■

Solution to 8.

$$y = \cosh(\ln x) \quad \text{use the fact } (\cosh(f(x)))' = f'(x)\sinh(f(x))$$

$$y' = \sinh(\ln x)\frac{1}{x}$$

$$y' = \frac{\sinh(\ln x)}{x}$$

■

Solution to 9.

$$\begin{aligned}
\lim_{x\to\infty}\left[\ln x-\sinh^{-1}x\right] &= \lim_{x\to\infty}\left[\ln x-\ln\left(x+\sqrt{1+x^2}\right)\right] \quad (\infty-\infty)\ \text{I.F.}\\
&= \lim_{x\to\infty}\ln\left(\frac{x}{x+\sqrt{1+x^2}}\right)\\
&= \lim_{x\to\infty}\ln\left(\frac{x}{x+\sqrt{x^2\left(\frac{1}{x^2}+1\right)}}\right)\\
&= \lim_{x\to\infty}\ln\left(\frac{x}{x\left(1+\sqrt{\frac{1}{x^2}+1}\right)}\right)\\
&= \lim_{x\to\infty}\ln\left(\frac{1}{1+\sqrt{\frac{1}{x^2}+1}}\right)\\
&= \ln\left(\frac{1}{2}\right)=-\ln 2.
\end{aligned}$$

■

Solution to 10.

Let $y = \left(\frac{\cos(3x)}{\cos(2x)}\right)^{\frac{1}{x^2}}$ $\ln y = \frac{\ln\left(\frac{\cos(3x)}{\cos(2x)}\right)}{x^2} = \frac{\ln(\cos(3x)) - \ln(\cos(2x))}{x^2}$

$$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{\ln(\cos(3x)) - \ln(\cos(2x))}{x^2} \quad \left(\frac{0}{0}\right) \text{ I.F.}$$

$$\overset{L.H.}{=} \lim_{x\to 0^+} \frac{\frac{-3\sin(3x)}{\cos(3x)} - \frac{-2\sin(2x)}{\cos(2x)}}{2x}$$

$$= \lim_{x\to 0^+} \frac{-3\tan(3x) + 2\tan(2x)}{2x} \quad \left(\frac{0}{0}\right) \text{ I.F.}$$

$$\overset{L.H.}{=} \lim_{x\to 0^+} \frac{-9\sec^2(3x) + 4\sec^2(2x)}{2} = \frac{-9+4}{2} = \frac{-5}{2}.$$

Hence

$$\lim_{x\to 0^+} \left(\frac{\cos(3x)}{\cos(2x)}\right)^{\frac{1}{x^2}} = e^{\frac{-5}{2}} = \frac{1}{\sqrt{e^5}}.$$

■

Solution to 11.

$$\lim_{x\to 0}\frac{\sin(4x)}{\sin(8x)} \overset{L.H.}{=} \lim_{x\to 0}\frac{4\cos(4x)}{8\cos(8x)}$$
$$= \frac{4\cos 0}{8\cos 0} = \frac{4}{8} = \frac{1}{2}.$$

■

Solution to 12.

$$\lim_{x\to 0}\frac{2^x-1}{x}\overset{L.H.}{=}\lim_{x\to 0}\frac{2^x\ln 2}{1}$$
$$=2^0\ln 2=\ln 2.$$

■

Solution to 13.

$$
\begin{aligned}
f''(\theta) &= \sin\theta + \cos\theta \\
f'(\theta) &= -\cos\theta + \sin\theta + C \qquad \text{use the fact } f'(0) = 4 \\
4 = f'(0) &= -\cos 0 + \sin 0 + C \\
4 &= -1 + C \\
C &= 5 \\
f'(\theta) &= -\cos\theta + \sin\theta + 5 \\
f(\theta) &= -\sin\theta - \cos\theta + 5\theta + D \qquad \text{use the fact } f(0) = 3 \\
3 = f(0) &= -\sin 0 - \cos 0 + 5(0) + D \\
D &= 4 \\
f(\theta) &= -\sin\theta - \cos\theta + 5\theta + 4
\end{aligned}
$$

■

Solution to 14.

$$\begin{aligned}
f''(x) &= 24x^2 + 6x + 4 \\
f'(x) &= 8x^3 + 3x^2 + 4x + C \\
f(x) &= 2x^4 + x^3 + 2x^2 + Cx + D && \text{use the fact } f(0) = 3 \\
3 = f(0) &= 2(0)^4 + (0)^3 + 2(0)^2 + C(0) + D \\
D &= 3 \\
f(x) &= 2x^4 + x^3 + 2x^2 + Cx + 3 && \text{use the fact } f(1) = 10 \\
10 = f(1) &= 2(1)^4 + (1)^3 + 2(1)^2 + C(1) + 3 \\
10 &= 8 + C \\
C &= 2 \\
f(x) &= 2x^4 + x^3 + 2x^2 + 2x + 3
\end{aligned}$$

■

Solution to 15.

Clearly $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \frac{1}{36} = \sum_{i=1}^{6} \frac{1}{i^2}$. ■

Solution to 16. $\sum_{j=0}^{4}\left(2^j+j^2\right)=(2^0+0^2)+(2^1+1^2)+(2^2+2^2)+(2^3+3^2)+(2^4+4^2)=1+3+8+17+32=61.$ ■

Solution to 17. $f(x) = x^2 + 1$ from $x = -1$ to $x = 2$. We have $n = 3$ $\Delta x = \frac{2-(-1)}{3} = 1$ $x_0 = -1, x_1 = -1 + 1 = 0, x_2 = 0 + 1 = 1, x_2 = 1 + 1 = 2$

$$\begin{aligned} R_3 &= [f(x_1) + f(x_2) + f(x_3)]\Delta x \\ &= [f(0) + f(1) + f(2)](1) \\ &= [(1) + (2) + (5)](1) = 8 \end{aligned}$$

■

Solution to 18.

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{x_i^*}{(x_i^*)^2+4}\Delta x=\int_1^3\frac{x}{x^2+4}\,dx$$

■

Solution to 19. Note that $f(x) = \frac{\sin x}{1+x^2}$ is an odd function and hence $\int_{-1}^{1} \frac{\sin x}{1+x^2}\, dx = 0$.

$$\int_{-2}^{1} f(x)\, dx = \int_{-2}^{-1} f(x)\, dx + \int_{-1}^{1} f(x)\, dx$$

$$30 = 10 + \int_{-1}^{1} f(x)\, dx$$

$$\int_{-1}^{1} f(x)\, dx = 30 - 10 = 20$$

$$\int_{-1}^{1} \left[\frac{3\sin x}{1+x^2} + 4f(x)\right] dx = 3\int_{-1}^{1} \frac{\sin x}{1+x^2}\, dx + 4\int_{-1}^{1} f(x)\, dx$$

$$= 3(0) + 4(20) = 80.$$

■

Solution to 20.

$$h(x) = \int_{1}^{e^x} \ln t \, dt \qquad \text{Use Fundamental Theory of Calculus .}$$

$$h'(x) = \ln(e^x)(e^x)'$$

$$h'(x) = xe^x.$$

■

Solution to 21. Note that $\ln(9^x) = x \ln 9$

$$\int_1^2 \frac{x^3 + 3x^6}{x^4}\,dx$$
$$= \int_1^2 \left[\frac{x^3}{x^4} + \frac{3x^6}{x^4}\right] dx$$
$$= \int_1^2 \left[\frac{1}{x} + 3x^2\right] dx$$
$$= \left[\ln|x| + x^3\right]_1^2$$
$$= (8 + \ln 2) - (1 + \ln 1) = 7 + \ln 2.$$

■

Solution to 22.

$$\int_1^{18} \sqrt{\frac{3}{x}}\, dx$$

$$= \int_1^{18} \frac{\sqrt{3}}{\sqrt{x}}\, dx$$

$$= \sqrt{3} \int_1^{18} x^{-1/2}\, dx$$

$$= \sqrt{3}\, \left[2\sqrt{x}\right]_1^{18}$$

$$= \sqrt{3}\left[2\sqrt{18} - 2\sqrt{1}\right]$$

$$= \sqrt{3}[6\sqrt{2} - 2] = 6\sqrt{6} - 2\sqrt{3}.$$

■

Solution to 23.

$$\int_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}} \frac{4}{\sqrt{1-x^2}}\,dx$$

$$= 4\int_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}} \frac{1}{\sqrt{1-x^2}}\,dx$$

Use the fact $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$.

$$= 4\left[\sin^{-1} x\right]_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}}$$

$$= 4\left[\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) - \sin^{-1}\left(\frac{1}{2}\right)\right]$$

$$= 4\left[\frac{\pi}{4} - \frac{\pi}{6}\right]$$

$$= \pi - \frac{2\pi}{3} = \frac{\pi}{3}.$$

■

Solution to 24.

$$\begin{aligned}\int (x+4)(2x+1)\,dx &= \int (2x^2+9x+4)\,dx \\ &= \frac{2}{3}x^3 + \frac{9}{2}x^2 + 4x + C\end{aligned}$$

■

Solution to 25. Note that $1 + \tan^2 \alpha = \sec^2 \alpha$

$$\begin{aligned}\int [1 + \tan^2 \alpha] \, d\alpha &= \int \sec^2 \alpha \, d\alpha \\ &= \tan \alpha + C.\end{aligned}$$

■

Solution to 26.

$$\int [10^x + x^{10}]\,dx$$

$$= \int 10^x\,dx + \int x^{10}\,dx \quad \text{Use the fact } \int a^x\,dx = \frac{a^x}{\ln a} + C\,.$$

$$= \frac{10^x}{\ln 10} + \frac{x^{11}}{11} + C.$$

■

Solution to 27. Note that $\left(\frac{1}{x}\right)' = (x^{-1})' = -x^{-2} = \frac{-1}{x^2}$.

$$\int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2}\,dx$$

$$= \int \sec^2\left(\frac{1}{x}\right)\frac{1}{x^2}\,dx \qquad \text{Use the fact } \int \sec^2(f(x))\,f'(x)\,dx = \tan(f(x)) + C \text{ .}$$

$$= -\int \sec^2\left(\frac{1}{x}\right)\frac{-1}{x^2}\,dx$$

$$= -\tan\left(\frac{1}{x}\right) + C.$$

■

Solution to 28. Note that $(1+\tan t)' = \sec^2 t = \frac{1}{\cos^2 t}$.

$$\int \frac{1}{\cos^2 t\sqrt{1+\tan t}}\, dt$$
$$= \int (1+\tan t)^{-1/2}\, \frac{1}{\cos^2 t} dt$$
$$= \int (1+\tan t)^{-1/2}\, \sec^2 t dt$$ Use the fact $\int (f(x))^n\, f'(x)\, dx = \frac{(f(x))^{n+1}}{n+1} + C$.
$$= \frac{(1+\tan t)^{1/2}}{1/2} + C$$
$$= 2\sqrt{1+\tan t} + C.$$

■

Solution to 29. Note that $\sin(2x) = 2\sin x\cos x$, $\left(1+\cos^2 x\right)' = -2\sin x\cos x$.

$$\int \frac{\sin(2x)}{1+\cos^2 x}\,dx$$

$$= \int \frac{2\sin x\cos x}{1+\cos^2 x}\,dx$$ Use the fact $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$.

$$= -\int \frac{-2\sin x\cos x}{1+\cos^2 x}\,dx$$ Use the fact $|1+\cos^2 x| = 1+\cos^2 x$.

$$= -\ln\left(1+\cos^2 x\right) + C.$$

■

Solution to 30. Note that $(\cos x)' = -\sin x$.

$$\int \frac{\sin x}{1+\cos^2 x}\,dx$$

$$= \int \frac{\sin x}{1+(\cos x)^2}\,dx \qquad \text{Use the fact } \int \frac{f'(x)}{1+[f(x)]^2}\,dx = \tan^{-1}(f(x)) + C\,.$$

$$= -\int \frac{-\sin x}{1+(\cos x)^2}\,dx$$

$$= -\tan^{-1}(\cos x) + C.$$

■

Solution to 31. Note that $\left(\sin^{-1} x\right)' = \frac{1}{\sqrt{1-x^2}}$.

$$\begin{aligned}
&\int \frac{1}{\sqrt{1-x^2}\sin^{-1} x}\,dx \\
&= \int \frac{\frac{1}{\sqrt{1-x^2}}}{\sin^{-1} x}\,dx \qquad \text{Use the fact } \int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C\ . \\
&= \ln|\sin^{-1} x| + C.
\end{aligned}$$

■

Solution to 32. Note that $(5^x)' = 5^x \ln 5$.

$$\int 5^x \sin(5^x)\, dx$$

$$= \frac{1}{\ln 5} \int \sin(5^x)\, 5^x \ln 5\, dx \quad \text{Use the fact } \int \sin(f(x))\, dx = -\cos(f(x)) + C \text{ .}$$

$$= \frac{-\cos(5^x)}{\ln 5} + C.$$

■

Solution to 33.

$$\int \frac{x^2}{1+x^6}\,dx$$
$$= \int \frac{x^2}{1+(x^3)^2}\,dx$$
$$= \frac{1}{3}\int \frac{3x^2}{1+(x^3)^2}\,dx$$

Use the fact $\int \frac{f'(x)}{1+[f(x)]^2}\,dx = \tan^{-1}(f(x)) + C$.

$$= \tan^{-1}(x^3) + C.$$

∎

Solution to 34.

$$\int \sin x \sec^2(\cos x)\,dx$$

$$= \int \sec^2(\cos x)\ \sin x\,dx$$

$$= -\int \sec^2(\cos x) - \sin x\,dx \quad \text{Use the fact } \int \sec^2(f(x))\,f'(x)dx = \tan(f(x)) + C\ .$$

$$= -\tan(\cos x) + C.$$

■

Solution to 35.

$$\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}\,dx \qquad \text{Note that } (\tan x)' = \sec^2 x\ .$$

$$= \int \frac{\sec^2 x}{\sqrt{1-(\tan x)^2}}\,dx \qquad \text{Use the fact } \int \frac{f'(x)}{\sqrt{1-[f(x)]^2}}\,dx = \sin^{-1}(f(x)) + C\ .$$

$$= \sin^{-1}(\tan x) + C.$$

■

Solution to 36.

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx \qquad \text{Note that } (\sqrt{x})' = \frac{1}{2\sqrt{x}}\,.$$

$$= 2\int e^{\sqrt{x}}\frac{1}{2\sqrt{x}}\,dx \qquad \text{Use the fact } \int e^{f(x)}f'(x)dx = e^{f(x)} + C\,.$$

$$= 2e^{\sqrt{x}} + C.$$

Solution to 37.

$$\int x^3\sqrt{x^2+1}\,dx$$ Let $w=\sqrt{x^2+1} \Leftrightarrow w^2=x^2+1.$

$$=\int x^2\sqrt{x^2+1}\,x\,dx$$ $x^2=w^2-1$ $2x\,dx=2w\,dw \Rightarrow x\,dx=w\,dw$.

$$=\int (w^2-1)ww\,wd$$

$$=\int (w^4-w^2)\,dw$$

$$=\frac{1}{5}w^5-\frac{1}{3}w^3+C$$

$$=\frac{1}{5}(\sqrt{x^2+1})^5-\frac{1}{3}(\sqrt{x^2+1})^3+C$$

$$=\frac{1}{5}(x^2+1)^{5/2}-\frac{1}{3}(x^2+1)^{3/2}+C.$$

■

Solution to 38.

$$\int_0^1 \frac{1}{(1+\sqrt{x})^4}\,dx \qquad \text{Let } w = 1+\sqrt{x} \Leftrightarrow w-1 = \sqrt{x}.$$

$$= \int_0^1 \frac{1}{(1+\sqrt{x})^4}\,dx \qquad x = (w-1)^2 \Leftrightarrow dx = 2(w-1)\,dw\ .$$

$$= \int_0^1 \frac{1}{(1+\sqrt{x})^4}\,dx \qquad x = 0 \Rightarrow w = 1,\ x = 1 \Rightarrow w = 2\ .$$

$$= \int_1^2 \frac{1}{(w)^4}\,2(w-1)\,dw \quad = \int_1^2 2w^{-4}(w-1)\,dw = \int_1^2 (2w^{-3}-2w^{-4})\,dw$$

$$= \left[\frac{2}{-2}w^{-2} - \frac{2}{-3}w^{-3}\right]_1^2 = \left[\frac{-1}{w^2} + \frac{2}{3w^3}\right]_1^2 = [\frac{-1}{4} + \frac{2}{24}] - [\frac{-1}{1} + \frac{2}{3}] = \frac{1}{6}.$$

■

Solution to 39.

$$\int \frac{1+x}{1+x^2}\,dx$$ Use $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$.

$$= \int \frac{1}{1+x^2} + \frac{x}{1+x^2}\,dx$$ Use $\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$.

$$= \int \frac{1}{1+x^2}\,dx + \frac{1}{2}\int \frac{2x}{1+x^2}\,dx$$ $\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + C$.

$$= \tan^{-1} x + \frac{1}{2}\ln(1+x^2) + C$$

$$= \tan^{-1} x + \ln(\sqrt{1+x^2}) + C$$

■

Solution to 40.

$$\int \frac{3^x}{1+3^x}\,dx \qquad \text{Note}(1+3^x)' = 3^x \ln 3.$$

$$= \frac{1}{\ln 3}\int \frac{3^x \ln 3}{1+3^x}\,dx \qquad \int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$$

$$= \frac{1}{\ln 3}\ln(1+3^x) + C$$

$$= \frac{\ln(1+3^x)}{\ln 3} + C.$$

■

Solution to 41.

$$\int \frac{3^x}{1+3^{2x}}\,dx \qquad \text{Note}(1+3^x)' = 3^x \ln 3 \text{ and } 3^{2x} = (3^x)^2.$$

$$= \frac{1}{\ln 3}\int \frac{3^x \ln 3}{1+(3^x)^2}\,dx \qquad \int \frac{f'(x)}{1+[f(x)]^2}\,dx = \tan^{-1}(f(x)) + C$$

$$= \frac{1}{\ln 3}\tan^{-1}(3^x) + C$$

$$= \frac{\tan^{-1}(3^x)}{\ln 3} + C.$$